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On the systematics of elementary particles

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Introduction

One feature of the present phenomenological theory of elementary particles seems particularly strange in view of our earlier knowledge, that is the assignment in the astonishingly successful scheme of Gell-Mann and Nishijima of whole as well as half integer isotopic spin both to fermions and bosons. Although we are doubtlessly very far from a fundamental theory of elementary particles it is, of course, important to see how much we can obtain of simplicity in anyone approximately isolated domain of experience without hurting against experimental facts. From this point of view some physicists have tried to regard the greater part of the strange particles and also the π-mesons as compounds between nucleons and a single kind of strange particles as K-mesons (Goldhaber) or Λ-particles (Sakata). Other physicists (Pais, Gell-Mann, Schwinger and others) have tried to regard all the strange particles together with the nucleons, the π-mesons and in some cases also the leptons as so to say elementary representatives of a larger group than that corresponding to isotopic spin.

In the following a tentative scheme is proposed which has something in common with each of the ways just mentioned but would seem to me to be a more natural generalisation of the isotopic spin scheme of Kemmer, which has been so successful as a basis for the description of the behaviour of nucleons and π-mesons.

1. A proposed scheme for barions and mesons

The ordinary charge-symmetric theory of the π-meson-nucleon interaction may, as is well-known, be based on an action integral, the fermion part of which is bilinear in the nucleon field

$$N=\begin{pmatrix} n \\ p \end{pmatrix}, \tag{1}$$

to which corresponds a charge matrix

$$Q_N=\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

whereby

$$Q_N = \frac{U}{2} + T_3, \tag{3}$$

with U (the hypercharge) equal to 1 and $T_3 = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$. To this comes the pseudoscalar π-meson field, which may be written as a two-row, isotopic spin matrix[1]

$$\Pi = \begin{pmatrix} -\pi^0, & \pi^- \\ \pi^+, & \pi^0 \end{pmatrix}, \tag{4}$$

the charge character of which (indicated by the notations) corresponds to the commutation relation

$$[Q_N, \Pi] = \begin{pmatrix} 0, & -\pi^- \\ \pi^+, & 0 \end{pmatrix}. \tag{5}$$

Apart from a field proportional to the unit matrix, (4) represents the most general field of the type in question. In the charge-symmetric theory π is regarded as an entity, i.e. its interaction with the nucleon field contains only one interaction constant for each type of interaction. That the mass difference between $\pi^\pm$ and π^0 is comparable with the electron mass is favouring this kind of treatment, which amounts to the assumption that the theory, as far as electromagnetic and weak interactions may be neglected, is invariant under general isotopic spin rotations, whereby

$$N' = T N, \tag{6}$$

T being a general isotopic spin rotation matrix.

Now, the simplest expansion of this scheme, so as to include "strangeness" would seem to be the addition of one more spin $\frac{1}{2}$ fermion field of zero charge and zero isotopic spin to N and replacing Π by a similarly enlarged matrix. We put, thus, for the barion field

$$\psi_B = \begin{pmatrix} \Lambda \\ n \\ p \end{pmatrix} \equiv \begin{pmatrix} \Lambda \\ N \end{pmatrix} \tag{7}$$

and for the corresponding meson field

$$F = \begin{pmatrix} X & \bar{K}^0 & K^- \\ K^0 & -\pi^0 & \pi^- \\ K^+ & \pi^+ & \pi^0 \end{pmatrix} \equiv \begin{pmatrix} X & \bar{K} \\ K & \Pi \end{pmatrix}, \tag{8}$$

their transformation properties under an isotopic spin transformation $\theta = \begin{pmatrix} 1 & 0 \\ 0 & T \end{pmatrix}$ being

$$\psi'_B = \theta\, \psi_B = \begin{pmatrix} \Lambda \\ T N \end{pmatrix} \tag{9}$$

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[1] This is equivalent to the usual expression $\tau_1 \pi_1 + \tau_2 \pi_2 + \tau_3 \pi_3$, where τ_1, τ_2, τ_3 are isotopic spin Pauli matrices, whereby $\pi^0 = -\pi_3$, $\pi^+ = \pi_1 + i\pi_2$, $\pi^- = \pi_1 - i\pi_2$.

and

$$F' = \theta F \theta^{-1} = \begin{pmatrix} X & \bar{K} T^{-1} \\ T K & T \Pi T^{-1} \end{pmatrix}. \tag{10}$$

Further we put

$$Q_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 \\ 0 & Q_N \end{pmatrix} \tag{11}$$

for the baryon charge matrix and get

$$[Q_B, \mathrm{F}] = \begin{pmatrix} 0, & 0, & -K^- \\ 0, & 0, & -\pi^- \\ K^+ & \pi^+ & 0 \end{pmatrix}, \tag{12}$$

which togeter with (9), (10) and (11) justifies the notations chosen, which should, however, not be taken too literally, since we are probably far from having found the correct action integral and since, even if this were the case, renormalisation would be required.

In favour of the scheme in question it may be pointed out that, apart from a field proportional to the unit matrix, the expression for F is a general hermitian three-row matrix and that it contains fields of the charge and transformation character of all the known bosonic mesons and on top of that only a single extra field X corresponding to a still unknown neutral boson of isotopic spin 0.

If, further, as usual, we define the hypercharge by means of (3), it follows that U for the baryons is represented by the diagonal matrix

$$U_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{13}$$

and that the commutation expression

$$[U_B, F] = \begin{pmatrix} 0, & -\bar{K}^0 & -K^- \\ K^0 & 0 & 0 \\ K^+ & 0 & 0 \end{pmatrix} \tag{14}$$

agrees with the usual hypercharge assignment to the π's and the K's. Further it follows that F acting on ψ_B produces a new ψ corresponding to the same charge- and hypercharge-values and that the product of two F-matrices is an F-matrix of the same character. We may say that F is a Heisenberg matrix, where charge and hypercharge are quantum numbers.

In the scheme in question there is no immediate place for the Σ's, and the Ξ's, so we must try to regard them as a kind of compound particles[1]. In this respect it looks promising that such simple compounds between a Λ-particle and a meson as (Π, Λ) and $(\bar{K}, \Lambda)$ have the correct values of charge and isotopic spin, the former forming an isotopic spin triplet like Σ and the latter an isotopic spin doublet of one negatively charged particle and one neutral particle as is assumed for Ξ. According to (3) the hypercharge assignment will then automatically be the same as in the Gell-Mann-Nishijima scheme. In fact, these compounds agree with those proposed by Sakata, if Π and $\bar{K}$ are replaced by the compounds $(\bar{N}, N)$ and $(\bar{\Lambda}, N)$ respectively according to his assumptions [1]. The only further compound of this simple type is (K, Λ), which would have the same charge and isotopic spin (and hypercharge) as the nucleons, so that, if it exists, one would assume that it would go into N by a fast electromagnetic or π-mesonic reaction.

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[1] Since there is no conservation law for the boson number the very concept of a compound system between a boson und a fermion ought not to be taken very literally but rather as an indication of the possibility of having "elementary excitations" of the general character of Σ's and Ξ's without any further extension of the basic fermion field.

The main objection against the proposed scheme is certainly the big mass difference between K's and π's, which makes a first approximation, where Λ and N and also K and π are simply regarded as different states of one fermion and one boson, each of a single mass, appear very doubtful. On the other hand, one may think of the baryons and the heavy bosons as forming in first approximation a simple mass spectrum, the states of which are determined by the total isotopic spin. It may be mentioned that within the accuracy of about ten times the electron mass m_e, one may, in fact, express the masses of Λ and N on the one side and those of π and K on the other side as simple functions of the isotopic spin, using $\frac{2\hbar c}{e^2}\, m_e$ as mass unit. Due to the very small number of applications and the limited accuracy such formulae could at present hardly be taken seriously, but may be kept in mind when further material becomes available, i.e. if a new meson were discovered, which could be the missing X in the expression (8) for F.

2. A proposed lepton scheme

There is an interesting similarity between the baryon scheme proposed here and what is known about leptons. As is well-known, it has been difficult to arrange the leptons according to isotopic spin and hypercharge and some physicists have completely denied the significance of these concepts for the leptons. On the other hand, a conservation law for the lepton number, in analogy to the baryon number conservation law, would not seem to contradict any present evidence, even if it is hardly yet an established law. Now, we are used to associate zero isotopic spin with a state of zero charge. But, on account of the relation (3) between charge, hypercharge and isotopic spin, according to which there is no unique correlation between charge and isotopic spin, a charged particle of isotopic spin zero is not excluded.

Thus, in the case of leptons we have three fundamental particles (with antiparticles), the negatively charged μ-meson, the negatively charged electron and the uncharged neutrino, which may be compared to Λ, n and p, each of them having just one unit charge less than the corresponding baryon. Moreover, e and ν have, like n and p, the same mass when we neglect masses of the order of magnitude m_e, while μ, like Λ, has a mass larger than that of the two other particles of the order of magnitude of the π-meson mass. We may thus form a ψ_L for the leptons, given by

$$\psi_L = \begin{pmatrix} \mu \\ e \\ \nu \end{pmatrix}, \tag{15}$$

analogous to the ψ_B given by (7), where μ forms an isotopic spin singlet and e, ν an isotopic spin doublet, T_3 being given by the diagonal matrix

$$T_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}. \tag{16}$$

We have hereby assumed that μ^-, e^- are particles and, thus, μ^+, e^+ antiparticles. This assumption is necessary if we let the charge (and also the mass) change in the same way as in the baryon case, when we pass from the top to the botton of ψ_L. Therefore we see that the processes $p \rightarrow e^+ + \gamma$, $n \rightarrow \nu + \gamma$, which are forbidden by the law of conservation of baryon number, are in this scheme already forbidden by the more general law of conservation of fermions. On the other hand, a number of processes, such as $n \rightarrow e^- + \pi^+$, which are forbidden on the former law, would here contradict the law of conservation of isotopic spin not generally conserved in weak processes, while ordinary β-processes but not μ-decay would conform to the latter law. Such considerations are, however, of a rather formal nature as long as we have no closer knowledge of basic interactions including leptons.[1]

However, the great success of the universal V–A theory for weak interaction [2] — as pointed out from many sides — has strengthened the belief in a vector boson interaction responsible for all such interactions, the field in question corresponding to charged bosons, no weak transitions without charge exchange being known — to which comes the violation of isotopic spin conservation in these transitions. Together with the above considerations this suggests the introduction of the following three-row boson field matrix

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[1] After having been informed that professor Marshak was working on similar lines I sent him a copy of the above, whereupon he kindly sent me a copy of the summary of a paper by him, Gamba and Okubo to be given at the annual meeting of the National Academy of Sciences in April. In the meantime the considerations following above had occurred to me. According to the summary in question these considerations are included in the paper as a part of a much fuller treatment of the subject, the whole being based on a symmetry principle $\Lambda \rightleftarrows \mu$, $n \rightleftarrows e$, $p \rightleftarrows \nu$, which would seem to agree with the above in that the triplets (Λ, n, p) and (μ, e, ν) are treated on the same footing as far as weak interactions are concerned. The treatment of the heavy particles differs, however, from the present considerations since the π and K mesons are regarded as compounds of Λ, n, p as in the theory of Sakata. As to fermions Marshak *et al.* state that they have derived a unified relation between charge, the third component of isotopic spin, the strangeness and the baryon (or lepton) charge. This is a further interesting point, which had evaded me, having used the hypercharge instead of the strangeness. In fact, it follows from the T_3 assignment for the leptons proposed above that for leptons as well as for baryons

$$Q = \frac{N_B - N_L + S}{2} + T_3,$$

where N_B and N_L are the baryon and lepton numbers, respectively and the strangeness S has the same value for corresponding baryons and leptons. I guess this is their unified relation, the simplicity of which would seem to give a further argument for the T_3-values assigned to the leptons.

$$\Phi = \begin{pmatrix} 0 & 0 & \Phi^- \\ 0 & 0 & \Phi^- \\ \Phi^+ & \Phi^+ & 0 \end{pmatrix}, \quad \Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4), \tag{17}$$

which, apart from a constant factor, gives rise to the following terms in the lagrangian density

$$(\bar{\Lambda}\gamma a p + \bar{n}\gamma a p)\,\Phi^- + (\bar{p}\gamma a \Lambda + \bar{p}\gamma a n)\,\Phi^+ + \\ + (\bar{\mu}\gamma a \nu + \bar{e}\gamma a \nu)\,\Phi^- + (\bar{\nu}\gamma a \mu + \bar{\nu}\gamma a e)\,\Phi^+,$$

where $\gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ being Dirac matrices and a the Feynman notation for the projection operator characteristic of the V–A theory.

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